

Title: What's Your Angle?

Brief Overview:

Students will learn the basic draw and measure features of Geometer's Sketchpad using precise, step by step instructions. The students will use Sketchpad to draw and measure the angles of polygons and calculate the sum of their interior angles. The students will then complete an activity that relates the number of triangles drawn from one vertex of a polygon with the sum of its interior angles.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will read and execute written directions to Geometer's Sketchpad, and they will write about their results. They also will derive a formula for the sum of the interior angles of a polygon through experimentation using Geometer's Sketchpad. Last of all, students will use reasoning skills to derive a formula and apply it.

- **Number and Operation**

Students will apply basic number operations to solve a formula.

- **Patterns, Functions, and Algebra**

Students will be able to use tables as tools to interpret expressions.

- **Geometry and Spatial Sense**

Students will draw and analyze polygons using the technology of Geometer's Sketchpad.

- **Measurement**

Students will measure angles of a polygon using the technology of Geometer's Sketchpad.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**

Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

Geometry, Measurement and Reasoning

- **2.1.1**

Students will describe the characteristics of geometric figures and will construct or draw geometric figures using technology and tools.

- **2.1.4**

Students will validate properties of geometric figures using appropriate tools and technology.

- **2.2.3**

Students will identify and use inductive and deductive reasoning.

- **2.3.1**

Students will use algebraic and geometric properties to measure indirectly.

Grade/Level:

Grades 8 – 12; Algebra and Geometry

Duration/Length:

1 to 2 class periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Basic number operations
- Knowledge of names and number of sides of different polygons
- Definition of a diagonal, equiangular, equilateral, complementary and supplementary

Student Outcomes:

Students will:

- learn the basic functions of Geometer's Sketchpad.
- draw polygons and calculate the sum of the interior angles.
- divide different polygons into triangles.
- calculate angles of any polygon, and find missing components.

Materials/Resources/Printed Materials:

- Computers with Geometer's Sketchpad installed
- Directions for each student
- Worksheets for each student

Development/Procedures:

During this unit, students may work with group members or partners. The teacher will give a warm-up that will introduce basic functions of Geometer's Sketchpad. Upon completion of the warm-up, the student will have a working knowledge of the functions needed to solve problems in Sketchpad.

Using Sketchpad, the student will draw and measure the interior angles of different polygons. Using inductive reasoning, the students will conclude that the sum of the interior angles for each polygon is a constant.

By dividing the polygons into different triangles, the student will discover that the sum of the interior angles depends on how many triangles may be formed from one vertex of the polygon.

Assessment:

Students will complete activity sheets for a grade.

Extension/Follow Up:

- Have students design an apartment using at least three different polygons using Sketchpad, i.e., give the students a square/rectangle as a layout and have them use three different polygons to create their floorplan for rooms within the apartment.
- Geometer's Sketchpad can be used to demonstrate rotations and translations of the polygons.
- Geometer's Sketchpad can be used to demonstrate The Pythagorean Theorem.

Authors:

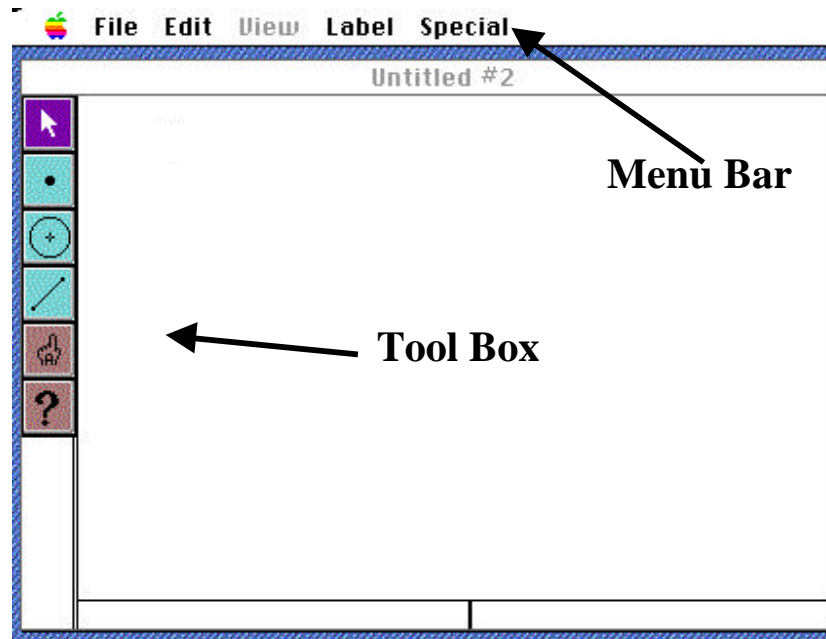
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WARM-UP ACTIVITY

An Introduction to Geometer's Sketchpad



GETTING STARTED

Your homescreen sketch will be ***Untitled #1***. Use your mouse to do the following:

- Click the ***point tool button*** (it will become purple)
- Use your mouse to place random points on the screen [*move & click, move & click*].
Note: the last point that you have created will be highlighted ~ you may click in any free space to remove this highlight.
- Click the ***hand tool button*** (your cursor will become a hand).
- Use your hand (cursor) to label the points on the screen.
Note: you must place the finger of the hand close enough to the point so that your hand turns black ~ sketchpad is now ready to label your points.

CLEARING YOUR SCREEN

(OPTION # 1)

Choose the ***arrow tool button***.
Click and drag a box around all items on your screen.
Press the ***delete key***.

(OPTION # 2)

Choose the ***arrow tool button***.
Choose ***select all*** from the menu
Press the ***delete key***.

CONSTRUCTING LINE SEGMENTS

- With a clear screen, choose the ***line segment tool button***.
- *Click* and *drag* to create some line segments on your screen.
- In a free space *click* and *drag* and hold down your ***shift*** key before you release the mouse to create the end of your line segment. [This action has Sketchpad create a line segment to the nearest 15 degrees.]

SELECTING OBJECTS

- Choose the ***arrow tool button***.
- Use your cursor to highlight one of your line segments.
- Press the ***shift*** key while highlighting and then select more than one object.

CONSTRUCTING MIDPOINTS

(With more than one segment highlighted)

- Choose “***construct***” from the file menu and then “***point at midpoint.***”
[Sketchpad has now constructed the midpoint of every segment that you had highlighted.]

CONSTRUCTING CIRCLES

- Clear your screen.
- Choose the ***circle tool button***. (Your cursor will look like the tool button.)
- *Click* and *drag* to construct a circle.
- Click in any free space to remove the handles from your circle.
[Your circle will now have a center point and one point on the circle that determines the radius.] Choose the ***hand tool button*** and label your circle at its center.
- Choose the ***point tool button*** and create any point outside of the circle.
- Choose the ***arrow tool button***, press the ***shift*** key to highlight the center of the circle and the point outside of the circle.
- From the menu choose ***CONSTRUCT*** and then ***SEGMENT***.

You should have a circle and a line segment on your screen.

CONSTRUCTING POLYGONS

(Option #1)

- Choose the ***line segment tool button***.
- *Click* and *drag* while connecting segments.

(Option #2)

- Choose the ***point tool button***.
- Place three points or more on your screen.
- Hold down the ***shift*** key and select all three points.
- From the menu bar choose ***construct*** and ***segment***.

Teacher Direction Sheet

Read and perform the following directions:

TO DRAW A POLYGON:

1. **Highlight the point** on the left **Tool Box**.
2. While holding down your **shift key**, use your mouse to randomly place three dots in the middle of your screen.
3. Go to the **Menu bar** at the top of your screen, highlight **CONSTRUCT** and arrow down to **segment**.
4. Go to the left **Tool Box** and highlight the **hand**. Click once on each vertex to label them. (*Note: you may move your labels by clicking and dragging them appropriately).

TO MEASURE THE ANGLES:

1. Highlight the **arrow key** in your left **Tool Box**.
2. Holding your **shift key** down, highlight **points A, B, and C** in that order.
3. Go to the **Menu bar**, highlight **MEASURE** and arrow down to **angle**.
(Notice that at the top left of your screen is the angle and its measure)
4. **Click** anywhere on the **empty screen** to eliminate the highlights.
5. **Repeat steps 5, 6, and 7** for angle **BCA** and angle **CAB**.
6. Highlight **MEASURE** on the **Menu bar** and arrow down to **calculate**. At the top left of your screen **highlight the measure of the first angle**, then **press plus on your calculator**. At the top left of your screen, highlight the **measure of the second angle**, then **press plus on your calculator**. **Repeat this process for the third angle**, then go to your calculator and **press the OK button**.
7. Holding down your **shift key**, **highlight all four calculations**. At the top **Menu bar**, highlight **MEASURE** and arrow down to **tabulate**.
8. **Click and drag on any vertex**. Notice how the measures of the angles change. **Double click on the chart** to add the new dimensions to your chart.
9. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

10. Perform the same procedure placing **four points** randomly in the middle of your screen.
11. Repeat steps **1 through 6** to find the **measures of all four angles and their sum**.
12. **Create a new chart** for this figure as directed in **step 7**.
13. **Click and drag on any vertex**. Add these new dimensions to your chart as directed in **step 8**.
14. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

15. Perform the same procedure placing **five points** randomly in the middle of your screen.
16. Repeat steps **1 through 6** to find the **measures of all five angles and their sum**.
17. **Create a new chart** for this figure as directed in **step 7**.
18. **Click and drag on any vertex**. Add these new dimensions to your chart as directed in **step 8**.
19. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

20. Perform the same procedure placing **six points** randomly in the middle of your screen.
21. Repeat steps **1 through 6** to find the **measure of all six angles and their sum**.
22. **Create a new chart** for this figure as directed in **step 7**.
23. **Click and drag on any vertex**. Add these new dimensions to your chart as directed in **step 8**.
24. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

Student Direction Sheet

Read and perform the following directions:

TO DRAW A POLYGON:

1. **Highlight the point** on the left **Tool Box**.
2. While holding down your **shift key**, use your mouse to randomly place three dots in the middle of your screen.
3. Go to the **Menu bar** at the top of your screen, highlight **CONSTRUCT** and arrow down to **segment**.
4. Go to the left **Tool Box** and highlight the **hand**. Click once on each vertex to label them. (*Note: you may move your labels by clicking and dragging them appropriately).

TO MEASURE THE ANGLES:

5. Highlight the **arrow key** in your left **Tool Box**.
6. Holding your **shift key** down, highlight **points A, B, and C** in that order.
7. Go to the **Menu bar**, highlight **MEASURE** and arrow down to **angle**.
(Notice that at the top left of your screen is the angle and its measure)
8. **Click** anywhere on the **empty screen** to eliminate the highlights.
9. **Repeat steps 5, 6, and 7** for angle **BCA** and angle **CAB**.
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11. Holding down your **shift key**, **highlight all four calculations**. At the top **Menu bar**, highlight **MEASURE** and arrow down to **tabulate**.
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Double click on the chart to add the new dimensions to your chart.
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23. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

24. Perform the same procedure placing **six points** randomly in the middle of your screen.
25. Repeat steps **1 through 6** to find the **measure of all six angles and their sum**.
26. **Create a new chart** for this figure as directed in **step 7**.
27. **Click and drag on any vertex**. Add these new dimensions to your chart as directed in **step 8**.
28. **Looking at your chart, what remains constant?**

CLEAR YOUR SCREEN

NAME _____

STUDENT ACTIVITY SHEET #1

DIRECTIONS: Draw a picture of your original polygon in the space provided. Fill in the chart for each polygon with the measures that you found using the Geometer's Sketchpad.

TRIANGLE:

	1 st	2 nd	3 rd	4 th
m∠ABC				
m∠BCA				
m∠CAB				
m∠ABC + m∠BCA + m∠CAB				

What remains constant? _____

QUADRILATERAL:

	1 st	2 nd	3 rd	4 th
m∠ABC				
m∠BCD				
m∠CDA				
m∠DAB				
m∠ABC + m∠BCD + m∠CDA + m∠DAB				

What remains constant? _____

PENTAGON:

	1 st	2 nd	3 rd	4 th
$m\angle ABC$				
$m\angle BCD$				
$m\angle CDE$				
$m\angle DEA$				
$m\angle EAB$				
$m\angle ABC + m\angle BCD +$ $m\angle CDE + m\angle DEA + m\angle EAB$				

What remains constant? _____

HEXAGON:

	1 st	2 nd	3 rd	4 th
$m\angle ABC$				
$m\angle BCD$				
$m\angle CDE$				
$m\angle DEF$				
$m\angle EFA$				
$m\angle FAB$				
$m\angle ABC + m\angle BCD +$ $m\angle CDE + m\angle DEF + m\angle EFA$ $+ m\angle FAB$				

What remains constant? _____

NAME _____

STUDENT ACTIVITY SHEET #1 ANSWER KEY

(**Note to teacher – answers will vary)

DIRECTIONS: Draw a picture of your original polygon in the space provided. Fill in the chart for each polygon with the measures that you found using the Geometer's Sketchpad.

TRIANGLE:

	1 st	2 nd	3 rd	4 th
m∠ABC				
m∠BCA				
m∠CAB				
m∠ABC + m∠BCA + m∠CAB	<i>180°</i>	<i>180°</i>	<i>180°</i>	<i>180°</i>

What remains constant? ***The sum of a triangle is always 180°.***

QUADRILATERAL:

	1 st	2 nd	3 rd	4 th
m∠ABC				
m∠BCD				
m∠CDA				
m∠DAB				
m∠ABC + m∠BCD + m∠CDA + m∠DAB	<i>360°</i>	<i>360°</i>	<i>360°</i>	<i>360°</i>

What remains constant? ***The sum of a quadrilateral is always 360°.***

PENTAGON:

	1 st	2 nd	3 rd	4 th
$m\angle ABC$				
$m\angle BCD$				
$m\angle CDE$				
$m\angle DEA$				
$m\angle EAB$				
$m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEA + m\angle EAB$	540°	540°	540°	540°

What remains constant? *The sum of a pentagon is always 540° .*

HEXAGON:

	1 st	2 nd	3 rd	4 th
$m\angle ABC$				
$m\angle BCD$				
$m\angle CDE$				
$m\angle DEF$				
$m\angle EFA$				
$m\angle FAB$				
$m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEF + m\angle EFA + m\angle FAB$	720°	720°	720°	720°

What remains constant? *The sum of a hexagon is always 720° .*

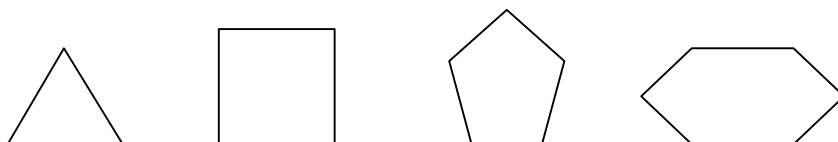
NAME _____

STUDENT ACTIVITY SHEET #2

Directions: Draw a quadrilateral, pentagon, and hexagon. From one vertex, highlight the vertex and one other non-consecutive vertex. Draw a segment. Clear your highlight by clicking anywhere on the empty screen. Continue highlighting the original vertex and other nonconsecutive vertices until the interior is divided into triangles.

Complete the chart below using your drawings from the Geometer's Sketchpad. Make sure to draw in your diagonals on each figure at the top of the column.

1.



# of sides				
# of triangles				
Difference				

2. What is the difference between the number of sides of a polygon and the number of triangles formed by the diagonals?
3. How many triangles can be formed from a polygon that has n number of sides?
4. Explain the rule relating the number of triangles within a polygon and the sum of its interior angles?

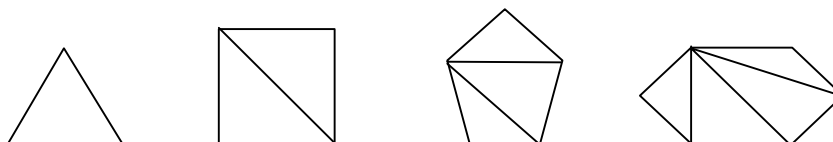
NAME _____

STUDENT ACTIVITY SHEET #2 ANSWER KEY

Directions: Draw a quadrilateral, pentagon, and hexagon. From one vertex, highlight the vertex and one other non-consecutive vertex. Draw a segment. Clear your highlight by clicking anywhere on the empty screen. Continue highlighting the original vertex and other nonconsecutive vertices until the interior is divided into triangles.

Complete the chart below using your drawings from the Geometer's Sketchpad. Make sure to draw in your diagonals on each figure at the top of the column.

1.



(Diagonals may vary)

# of sides	3	4	5	6
# of triangles	1	2	3	4
Difference	2	2	2	2

2. What is the difference between the number of sides of a polygon and the number of triangles formed by the diagonals?

The difference is always two.

3. How many triangles can be formed from a polygon that has n number of sides?

The number of sides minus two $[(n - 2)]$.

4. Explain the rule relating the number of triangles within a polygon and the sum of its interior angles??

The number of sides minus two, multiplied by $180^\circ [(n-2)*180]$.

Assessment

Teacher's Guide

Introduction

This performance assessment is intended to be used after the unit “What’s your Angle?” has been completed. The assessment consists of a combination of selected response items and brief constructed response items.

Objectives Covered

After the unit has been given, students should be able to do the following:

- Identify the basic functions, toolbar and menu of Geometer’s Sketchpad
- Construct polygons and calculate the sum of their interior angles
- Find missing components of angles in polygons
- Divide polygons into triangles

This assessment is to be given as an independent activity. It is suggested that the student activities be graded and the performance assessment graded as a quiz.

Tools/Materials Needed for Assessment

- Pencil
- Copy of assessment
- Calculator (optional)

Administering the Assessment

Students should be allowed approximately 20 minutes to complete the assessment.

Performance Assessment
Math Assessment – “What’s Your Angle?”
Student Response Sheet

Name_____ Date_____ Period_____

A. Place a circle around the letter of the correct answer for numbers 1-6.

- 1) Polygon PENTA has four angles whose sum is 478° . What is the measure of the fifth angle?
a) 62 b) 108 c) 135 d) 242
- 2) The sum of the measures of the interior angles of a triangle is?
a) 90 b) 160 c) 180 d) 540
- 3) To measure an angle in Geometer’s Sketchpad you would:
a) use a protractor b) choose **measure** from the menu
c) choose the **hand** tool button d) use a compass
- 4) An octagon has how many sides?
a) 2 b) 4 c) 6 d) 8
- 5) The number of triangles formed from a single vertex by the diagonals of a decagon is
a) 7 b) 8 c) 10 d) 11
- 6) What determines the sum of the interior angles of a polygon?
a) number of sides b) length of its sides
c) direction it is facing d) none of these answers
- 7) In class you were given a hexagon with two angles that have measures of 130° , one angle measures 98° , another angle measures 76° , and one more angle that measures 137° . Your problem is to solve for the missing angle. Your partner was absent and needs to know how to solve the problem. Write the steps you would use to describe to him/her how you would solve the problem. Be sure to number each step.

Performance Assessment
Math Assessment – “What’s Your Angle?”
Answer Key

Name _____ Date _____ Period _____

A. Place a circle around the letter of the correct answer for numbers 1-6.

- 1) Polygon PENTA has four angles whose sum is 478° . What is the measure of the fifth angle?
a) **62** b) 108 c) 135 d) 242
- 2) The sum of the measures of the interior angles of a triangle is?
a) 90 b) 160 c) **180** d) 540
- 3) To measure an angle in Geometer’s Sketchpad you would:
a) use a protractor **b) choose measure from the menu**
c) choose the **hand** tool button d) use a compass
- 4) An octagon has how many sides?
a) 2 b) 4 c) 6 **d) 8**
- 5) The number of triangles formed from a single vertex by the diagonals of a decagon is
a) 7 **b) 8** c) 10 d) 11
- 6) What determines the sum of the interior angles of a polygon?
a) **number of sides** b) length of its sides
c) direction it is facing d) none of these answers
- 7) In class you were given a hexagon with two angles that have measures of 130° , one angle measures 98° , another angle measures 76° , and one more angle that measures 137° . Your problem is to solve for the missing angle. Your partner was absent and needs to know how to solve the problem. Write the steps you would use to describe to him/her how you would solve the problem. Be sure to number each step.

Step One: Calculate the sum of the interior angles of a hexagon by using the formula $(n-2)(180)^\circ$.

Step Two: Subtract the given angles from 720° .

Step Three: Conclude that the unknown angle measures 149° .

Scoring Rubric
Performance Assessment Item # 7.

- 4 – Student explains that the sum of the measures of the interior angles of a polygon must be solved for by using the formula $(n-2)(180)^\circ$. Student also identifies the number of sides to be six. Student then explains that the given angles must be subtracted from 720 giving the measurement of the unknown angle to be 149° .
- 3 - Student explains that the sum of the measures of the interior angles of a polygon must be solved for by using the formula $(n-2)(180)^\circ$. Student also identifies the number of sides to be six. Student then explains that the given angles must be subtracted from 720 but student calculates the unknown angle incorrectly.
- 2 - Student explains that the sum of the measures of the interior angles of a polygon must be solved for by using the formula $(n-2)(180)^\circ$. Student identifies the number of sides incorrectly. Student may or may not calculate the unknown angle correctly.
- 1 - Student uses an incorrect formula. Student may or may not identify the number of sides correctly. Student also attempts to calculate the measure of the unknown angle.
- 0 - Student makes no attempt to solve the problem.